NWFP PUBLIC SERVICE COMMISSION

Competitive Examination
For The Provincial Management Service (BPS-17)

PURE MATHEMATICS - PAPER-I

Time Allowed: **THREE** Hours | Maximum Marks: **100**

NOTE: 1. Attempt Any **FIVE** questions in all, selecting at least **TWO** questions from each **SECTION**.

2. Extra attempt of question or part will not be considered.

3. Candidate must draw two straight lines (==========) at the end of the answer to separate each question attempted in the Answer Book.

SECTION – A

Q.1. (a) Define integral domains and prove that every finite integral domain is a field.

(b) If L is a finite extension of K and M is a finite extension of L, then M is a finite extension of K with $[M:K]=[M:L][L:K]$

Q.2. (a) The Housing Department of the NWFP Government plane to undertake four housing projects and lists material requirements for each house in each of the project as follows:

	Project 1	Project 2	Project 3	Project 4
Paint (in 100 gallons)	1	2	1	1.5
Wood (in 10,000 cu ft)	3	4	2.5	2.5
Bricks (in millions)	1	2	1.5	1
Labour (in 1000 hrs)	10	10	9	8

If the supplier delivers 6,800 gallons paint, 1,420,000 cubic fts of wood, 64 millions bricks and 4,48,000 hours of labour, find the number of houses built for each project.

(b) State Cayley – Hamilton Theorem and verify the Theorem for the matrix M, if

$$M=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

Also find inverse of the matrix M if exist.

Q.3. (a) Define Cyclic Group.

Let G be a group and let $a \in G$ then $H=\{a^n / n \in Z\}$ is a subgroup of G.

(b) Prove that if $\phi : G \to G'$ is an isomorphism of G with G' and e is the identity of G, then $e\phi$ is the identity in G'

and also prove that $a^{-1}\phi = (a\phi)^{-1}$ for all $a \in G$

(Contd: on Page 2)

Q.4. (a) Let W be the subset of $\mathbb{R}^3$ defined by

$$W = \{x : x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad x_2 = 2x_1, \quad x_3 = 3x_1, \quad x_1 \text{ any real number} \}$$

verify that W is a subspace of $\mathbb{R}^3$.

(b) Let V be the vector space of (2×2) matrices, and let W be the subspace

$$W = \{A : A = \begin{bmatrix} 0 & a_{12} \\ a_{21} & 0 \end{bmatrix}, a_{12} \text{ and } a_{21} \text{ are real scalars}\}$$

Define matrices B_1, B_2 *and* B_3 in W by

$$B_1 = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \text{and} \quad B_3 = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix},$$

Show that the set $\{B_1, B_2, B_3\}$ is linearly independent and express B_3 as a linear combination of B_1 and B_2. Also show that $\{B_1, B_2\}$ is linearly independent set.

SECTION – B

Q.5. (a) Find the vector equation of the plane containing the points $A(0,1,1)$, $B(2,1,0)$, $C(-2,0,3)$

(i) in parametric form (ii) in scalar product form

(b) Find the equation of the tangent line to the curve $x = 2t + 4, \quad y = 8t^2 - 2t + 4$ at $t = 1$ without eliminating the parameter.

Q.6. (a) Find the equation of the surface $x^2 + y^2 + z^2 = 1$, in

(i) Cylindrical co-ordinate (ii) Spherical co-ordinate

(b) Find the equation of the plane whose points are equidistant from (2, -1, 1) and (3, 1, 5).

Q.7. (a) Find the total are length of the cardioid $r = 1 + \cos\theta$

(b) Find an equation of the sphere with centre at $(2, 1, -3)$ that is tangent to the plane $x - 3y + 2z = 4$.

Q.8. (a) Convert from Rectangular to Cylindrical and Spherical coordinates $\left(-5\sqrt{3}, 5, 0\right)$

(b) Find the distance from the point $P(1, 4, -3)$ to the line $L : x = 2 + t, \quad y = -1 - t, \quad z = 3t$

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