NWFP, PUBLIC SERVICE COMMISSION, PESHAWAR

COMPETITIVE EXAMINATION FOR PROVINCIAL MANAGEMENT SERVICE, 2008

PURE MATHEMATICS, PAPER-II

Time Allowed: **03 Hours** **Max. Marks: 100**

Instructions:

(i) Attempt FIVE questions in all, selecting at least TWO questions from each section.
(ii) Do not use any list of formulae.
(iii) All questions carry equal marks

SECTION-A

Q.1 (a) Evaluate the following:

(i) $(2e^x - e^{2x})^{1/x^2}$ as $x \to 0$ — 04

(ii) $(\frac{1}{x^2} - \cot^2 x)$ *as* $x \to 0$ — 06

(b) State & prove cauchy's Mean value theorem and also show that the equation. — 10
$6x^5+5x^4+4x^3+3x^2-5=0$ has at least one real root in (0, 1)

Q.2 (a) Find an approximate formula for the change in total surface area of a right circular cylinder if small changes are made in the base radius 'r' and the height 'h'. — 04

(b) Find the extreme values of $f(x, y) = 2(x-y)^2 - x^4 - y^4$ — 08

(c) Evaluate $\iint_E (x^2 + y^2)\,dx\,dy$ where E is the interior of one leaf of the four-leaved rese $r = \cos 2\theta$ — 08

Q.3 (a) If p<1 than show that $\int_1^{\infty} \frac{x^p dx}{1+x^2}$ converges — 06

(b) Determine whether the function 'f' defined below — 06
$f(x) = 1$, if x is irrational
$= 0$, if x is rational
is Riemann integrable on [0, 1]

(c) Show that $\int_0^{\pi} \frac{x \sin x}{(1+\cos^2 x)} dx = \frac{\pi^2}{4}$ — 08

Q.4 (a) Define a metric space. Let X = R be the set of all real numbers and let d: R X R $\to$ R be defined by $d(x_1, x_2) = |x_1 - x_2|$ then show that (R, d) is a metric space. — 06

(b) Prove that an open sphere in a metric space 'X' is an open set. — 06

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(c) Define Cauchy sequence and Bounded sequence. Also show that the sequence $\{S_n\}$ defined by $S_1 = 1$, $S_{n+1} = \frac{4+3S_n}{3+S_n}, n \in N$ is convergent and find its limit. 08

SECTION – B

Q.5 (a) Define analytic function and prove that the real and imaginary parts of an analytic function of a complex variable when expressed in polar form satisfy the equation 05

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{1}{r}\frac{\partial\Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2} = 0$$

(b) If $U = (x-1)^3 - 3xy^2 + 3y^2$ then determine 'V' so that U+iV is an analytic function of x + iy 05

(c) State & prove Demoiver's theorem for integral exponent. 10

Q.6 (a) Find the Fourier series of the function 12

$$f(x) = \frac{1}{1+a\cos x}, -\pi \le x \le \pi$$

$$0 < a < 1$$

(b) Expand f(x) = Sin x in a Fourier Cosine series in the interval $0 \le x \le \pi$ 08

Q.7 Evaluate the following by Residue theorem

(a) $$\int_{-\infty}^{\infty}\frac{x^2\,dx}{(x^2+1)^2(x^2+2x+2)}$$ 10

(b) $$\int_0^{2\pi}\frac{\cos 3x}{5-4\cos x}\,dx$$ 10