K.P.K,PUBLIC SERVICE COMMISSION,PESHAWAR

COMPETATIVE EXAMINATION FOR PROVINCIAL MANAGEMENT SERVICE,2010

APPLIED MATHEMATICS,PAPER-II

TIME: 3 hours **Max Marks:** 100

Note: Attempt only **FIVE** questions, selecting at least **ONE** question from each section.

Each part carries 10 marks.

SECTION A

Q.1 Solve any two of the following differential equations.

(i) $\frac{x+y-a}{x+y-b}\frac{dy}{dx} = \frac{x+y+a}{x+y+b}$

(ii) $x.Sin\left(\frac{y}{x}\right)dy = \left\{y.Sin\left(\frac{y}{x}\right) - x\right\}dx$

(iii) $(1+y^2)dy = (Tan^{-1}y - x)dx$

Q.2 (a) Under certain conditions, cane sugar is converted into dextrose at a rate which is proportional to the amount unconverted at any time. If out of 75 grams of sugar at t=0, 8 grams are converted during the first 3 minutes. Find the amount converted in 90 minutes.

(b) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} + y = \tan x$$

Q.3 (a) Solve the following partial differential equation.

$$y^2p - xyq = x(z-2y) \quad where\ p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}$$

(b) The variation of an elastic string is governed by the P.D.E. $\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}$.
The length of the string is π and the ends are fixed.The initial velocity is zero and the initial deflection is $U(x,0) = 2(Sinx + Sin3x)$.Find the deflection u(x,t) of the vibrating string for $t > 0$.

SECTION B

Q.4 (a) A covariant tensor has components $xy, 2y - z^2, xz$ in rectangular coordinates. Find its covariant components in spherical coordinates.

(b) If $(ds)^2 = r^2(d\theta)^2 + r^2 Sin^2\theta(d\varphi)^2$. Find the value of [22,1].

Q.5 (a) If A^{ij} are the cofactors of a^{ij} in a determinant Δ of order 3, then show that $a^{ij}A^{kj} = \Delta\delta_i^k$

(b) Prove any two of the following.

(i) $curl(grad\varphi) = 0$

(ii) $grad(div\ \vec{f}) = curl(curl\ \vec{f}) + \nabla^2\vec{f}$

(iii) $div(\vec{f} \times \vec{g}) = \vec{g}.curl\vec{f} - \vec{f}.curl\vec{g}$

SECTION C

Q.6 (a) Starting with $x_0 = 3$. Use Newton Raphson method to find a root of $x^3 - 3x - 5 = 0$, correct to 3 decimal places.

(b) Find by the method of Regula falsi a root of the equation.

$x^3 + x^2 - 3x - 3 = 0$, lying between 1 and 2.

Q.7 (a) Use the method of iteration to solve the equation $x = e^{-x}$ starting with $x = 1$ Perform 4 iterations up to 4 decimal places.

(b) Evaluate $\int_0^{\frac{\pi}{3}} \sqrt{1 - \frac{1}{3}Sin^2\theta}\ d\theta$, using Simpsons rule with 6 intervals, correct to 3 decimal places.

Q.8 (a) Solve the following system of equations by Jacobi's method.

$$4x + y + 3z = 17$$
$$x + 5y + z = 14$$
$$2x - y + 8z = 12$$

(b) Solve the following system of equations by using Gauss-Seidel method.

$$2x - y + 2z = 3$$
$$x + 3y + 3z = -1$$
$$x + 2y + 5z = 1$$

----------THE END----------