KHYBER PAKHTUNKHWA, PUBLIC SERVICE COMMISSION, PESHAWAR.

SUBJECT:- **COMPETITIVE EXAMINATION FOR THE POSTS OF PROVINCIAL MANAGEMENT SERVICES (BPS-17)**

(2010)

PURE MATHS (PAPER-II)

Time Allowed: 03 hours **Max: Marks: 100**

Instructions:
(i) Attempt FIVE Questions in all, Selecting at least THREE Questions from Section-A and TWO Questions from Section-B.
(ii) Do not use any list of formulae
(iii) All Questions carry equal marks

SECTION-A

I (a) For arbitrary real x and y, show that $|x+y| \le |x|+|y|$. When does equality hold? (10)

(b) If $f(x) = \int \frac{2\sin x - \sin 2x}{x^3}\, dx$, $x \neq 0$, then find $\lim_{x \to 0} f'(x)$. (10)

II (a) Use ~~Lagrange~~ Mean Value Theorem to show that $|\sin x - \sin y| \le |x-y|$ for any real numbers x and y. (10)

(b) For Beta function β, show that $\frac{\beta(m+1, n)}{\beta(m, n+1)} = \frac{m}{n}$ (10)

III (a) Find the area of the region lying between the curve $x^2(x^2+y^2) = a^2(y^2-x^2)$ and its asymptotes. (10)

(b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{2-x} xyz\, dz\, dy\, dx$. (10)

IV (a) If f_x, f_y, f_{yx} all exist in the neighbourhood of the point (a,b) and f_{yx} is continuous at (a,b), then show that f_{xy} also exists at (a,b) and $f_{xy} = f_{yx}$. (10)

(b) Prove that the function $f(x,y) = (|xy|)^{\frac{1}{2}}$ is not differentiable at origin, but that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ both exist at the origin (10)

P.T.O.

V (a) Define metric space. Let A be a subset of a metric space. Prove that interior of A denoted by A^o is the union of all open sets contained in A. (1

(b) Define continuity in metric space. If (X, d) and (Y, d') are two metric spaces, then prove that a function $f: X \longrightarrow Y$ is continuous on X iff the inverse image of each open subset of Y is open in X. (1

SECTION—B

VI (a) Show that the function $f(z) = |z|^2$ is not analytic in any domain (10)

(b) Determine the isolated singularities of each function: (5,5

(i) $f(z) = \dfrac{z^3-1}{z^5+1}$ (ii) $f(z) = \dfrac{4z^2+5z+3}{2z^4+z^3-13z^2+z-15}$.

VII (a) Using Cauchy Integral Theorem, Show that (10)

$$\int_\Gamma \frac{\cos z + \cosh\left(\frac{z}{2}\right)}{(z^2+16)(z^2-25)}\, dz,$$ where Γ is simple closed contour represented by circle $|z| = 2$.

(b) Expand in a Taylor Series the function f defined by $f(z) = \operatorname{Log} z = \operatorname{Log}|z| + i \operatorname{Arg} z \; (-\pi < \operatorname{Arg} z \leq \pi)$ about the point $z_0 = -1+i$. (1

VIII (a) Obtain the Laurant series expansion in power of z for the function defined by $f(z) = \dfrac{1}{(z-1)(z-3)}$. (10)

(b) Using Cauchy Residue Theorem, show that (10)

$$\int_C \frac{z^2+4}{(z-i)(z+i)}\, dz = 0,$$ where C is the circle $|z| = 1$ described in the positive direction.