KHYBER PAKHTUNKHWA, PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR THE POSTS OF PROVINCIAL MANAGEMENT SERVICES (BPS-17)

2013

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: 03 HOURS **MAX:MARKS: 100**

Note:- Attempt any Five Questions, selecting two questions from Section-A, ONE question from Section-B and two questions from Section-C. All questions carry equal marks.

SECTION-A

Q1. (a) Solve the differential equation $\left(x^2y^3 - \frac{1}{1+9x^2}\right)dx + x^3y^2dy = 0$

(b) The slope of the tangent line to a curve at the point (x, y) on the curve $6x^3\sqrt{x^4+9}$. If the point $(2, 250)$ lies on the curve, find an equation of the curve.

Q2. (a) Use the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} + y = \tan x \sec x$$

(b) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$

Q3. (a) Find the solution of partial differential equation

$$y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} + yu + xy$$

(b) Find the solution of the Cauchy Problem consisting of the partial differential equation $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ together with the initial condition $u(0, y) = \sin y$.

SECTION-B

Q4. (a) Write Newton's law in tensor form.

(b) Find g and g^{jk} corresponding to

$$ds^2 = 3(dx^1)^2 + 2(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^3$$

Q5. (a) Prove that for an orthogonal co-ordinate system

$$g_{11} = \frac{1}{g^{11}}, \quad g_{22} = \frac{1}{g^{22}}, \quad g_{33} = \frac{1}{g^{33}}$$

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(b) Prove that

(i) $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

(ii) $\nabla r^n = n r^{n-2} r$, where $r = xi + yj + zk$.

SECTION-C

Q6. (a) Find a positive real root of $e^x - 3x = 0$.

(b) Find the root of $x^3 - 3x - 3 = 0$ to four decimal places, using Newton Raphson method, that lies near $x = 2$.

Q7. (a) Using the method of Regula Falsi to find the positive real roots of $x^3 + x^2 - 3x - 3 = 0$

(b) Evaluate $\int_0^{\pi/2} \frac{\cos x}{1+x} dx$, using Simpson's rule with four intervals, correct to 3 decimal places.

Q8. (a) Solve the following System of equations by using Gauss-Seidel method

$$4x_1 - 2x_2 + x_3 = 12$$
$$2x_1 + 3x_2 - x_3 = 7$$
$$2x_1 - 2x_2 + 2x_3 = 8$$

(b) Solve the following system of equations by Jacobi's method

$$x_1 + 6x_2 + 2x_3 = 15$$
$$x_1 + x_2 - 6x_3 = -3$$
$$6x_1 + x_2 + x_3 = 9$$