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COMPETITIVE EXAMINATION FOR PROVINCIAL MANAGEMENT SERVICE, 2013

PURE MATHEMATICS, PAPER-II

Time Allowed: **03 Hours**
Max. Marks: 100

Instructions:

(i) Attempt **FIVE** questions in all, selecting at least **TWO** questions from each section.
(ii) Do not use any list of formulae.
(iii) All questions carry equal marks.

SECTION – A

Q1(a): Evaluate the following:

(i) $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{sin^2x}\right)$ (06)

(ii) $\lim_{x\to 1}\frac{x^x-x}{1-x+lnx}$ (06)

(b): Use Beta integral to evaluate $\int_0^{\pi/2} sin^4t cos^5t\, dt.$ (08)

Q2(a): Determine the area bounded by the curve whose equation is $y=\frac{1}{\sqrt{8-x}}$, the x-axis, y-axis and the asymptote x=8 (10)

(b): Find the extreme values of $f(x,y)=x^3+y^3-63(x+y)+12xy.$ (10)

Q3(a): Determine the convergence or divergence of the integral $\int_0^{\infty} e^{-x}x^5\, dx.$ (10)

(b): Derive Reduction Formula for $\int sin^n x\, dx$ and use it to evaluate $\int_0^{\pi/2} sin^9x\, dx.$ (10)

Q4(a): Prove that $\frac{\pi^2}{44}<\int_0^{\pi}\frac{x^3}{7+4cosx}\,dx<\frac{\pi^2}{12}.$ (10)

(b): Calculate by double integration, the volume generated by the revolution of the cardioids $r=a\,(1-\cos\theta)$ about its axis. (10)

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SECTION - B

Q5(a): Derive Cauchy-Riemann equation in the polar form. (10)

(b): If $\omega = \emptyset + i\varphi$ is an analytic function then determine $\emptyset$ where $\varphi = x^2 - y^2 + \frac{x}{x^2+y^2}$ (10)

Q6(a): Find the Fourier series of the function $f(x) = x - x^2$ where $-\pi < x < \pi$ (10)
and deduce $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\dots\dots\dots = \frac{\pi^2}{12}$.

(b): Expand $f(x) = \begin{cases} \frac{1}{4} - x & 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \frac{1}{2} < x < 1 \end{cases}$ as the Fourier series of sine terms. (10)

Q7: Evaluate the following by Residue theorem:

(i) $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}\, d\theta$ (10)

(ii) $\int_0^{\infty} \frac{dx}{1+x^6}$. (10)

Q8(a): Define a metric space. Let X=R be the set of all real numbers and let d: RxR→ R be define by $d(x_1,x_2) = |x_1 - x_2|$ then show that (R.d) is a metric space. (06)

(b): Prove that an open sphere in a metric space 'X' is an open set. (06)

(c): Define Cauchy sequence and Bounded sequence. Also show that the sequence $\{S_n\}$ define by $S_n = 1, S_n = \frac{4+3S_n}{3+S_n}$, $n \in N$ is convergent and find its limit. (08)
